ON SELF-SIMILAR MOTIONS IN THE THEORY OF THE NONSTATIONARY FILTRATION OF A GAS IN A POROUS MEDIUM

(OB AVTOMODEL'NYKH DVIZHENIIAKH V TEORII NESTATSIONARNOI FIL'TRATSII GAZA V PORISTOI SREDE)

PMM Vol.24, No.3, 1960, p. 558

A. M. PIRVERDIAN
(Baku)

(Received 10 March 1960)

The equation of planar isothermal filtration of a gas has the form [1]

$$\partial p / \partial t = a^2 \partial^2 p^2 / \partial x^2 \qquad (1)$$

For this equation there was found a series of self-similar solutions and, in particular, a solution of the form exp (αx_2) $f(x_1 \exp(\beta x_2))$, where x_1 and x_2 are independent variables [2]. The latter solution has been obtained on the basis of a special group of continuous invariant transformations $x_i + \xi_i (i = 1, 2)$, apart from the similarity transformations.

The derivation of the indicated self-similar solutions based only on the concepts of similarity theory (without transformations of the form $x_i + \xi_i$) is given below.

We will first consider the case in which the conditions for Equations (1) are given in the form [2]

$$p(x, -\infty) \equiv 0, \qquad p(0, t) = p_0 e^{\sigma t} \qquad (2)$$

Introducing the variable $u = p_0 e^{\sigma t}$, Equation (1) and the boundary conditions (2) take the form

$$u \frac{\partial p}{\partial u} = \left(\frac{a^2}{\sigma}\right) \frac{\partial^2 p^2}{\partial x^2}, \qquad p(x, 0) \equiv 0, \qquad p(0, u) = u \qquad (3)$$

From the equation and boundary conditions (3) it follows that the pressure p depends on three quantities x, u, a^2/σ whose dimensions are as follows:

$$[x] = L, \qquad [u] = [p], \qquad \left[\frac{a^2}{\sigma}\right] = [p]^{-1} L^2 \qquad (4)$$

According to the Π-theorem [3] we have $p = uf\ (a^2u/\sigma x)$ or, in the previous variables,

$$p = p_0 e^{\sigma t} f\left(\frac{x}{\sqrt{a^2 p_0 e^{\sigma t} \sigma^{-1}}}\right) \tag{5}$$

which coincides with the result [2] obtained in another way. The solution is obtained in an analogous manner when the initial pressure distribution is given in the form

$$p(x, 0) = p_0 e^{\beta x} \tag{6}$$

We introduce the variable $v = p_0 e^{\beta x}$; Equation (1) and Condition (6) take the form

$$\frac{\partial p}{\partial t} = a^2\beta^2 v^2 \frac{\partial^2 p^2}{\partial v^2}, \qquad p(v, 0) = v \tag{7}$$

The three independent quantities v, t, and $(a\beta)^2$ have the dimensions

$$[v] = [p], \qquad [t] = T, \qquad [(a\beta)^2] = [p]^{-1} T^{-1} \tag{8}$$

Hence, dimensional analysis gives

$$p = vf(a^2\beta^2 vt), \quad \text{or} \quad p = p_0 e^{\beta x} f(a^2\beta^2 p_0 e^{\beta x} t) \tag{9}$$

which also coincides with one of the results of the work of [2]. For the more general equation of filtration of a polytropic gas in a porous medium

$$\frac{\partial P}{\partial t} = b^2 \frac{\partial^2 P^n}{\partial x^2} \qquad \text{for} \quad P(x, -\infty) \equiv 0,\ P(0, t) = P_0 e^{\sigma t}; \quad \text{or} \quad \text{for} \quad P(x, 0) = P_0 e^{\beta x} \tag{10}$$

by means of the substitutions $u = P_0 e^{\sigma t}$ or $u = P_0 e^{\beta t}$, respectively, we obtain the solutions

$$P = P_0 e^{\sigma t} f\left(\frac{x}{\sqrt{b^2 P_0^{\,n-1} e^{(n-1)\sigma t} \sigma^{-1}}}\right) \quad \text{or} \quad P = P_0 e^{\beta x} f(b^2\beta^2 P_0^{\,n-1} e^{(n-1)\beta x} t) \tag{11}$$

From the examples cited it follows that for the solution of the problem new variables are chosen such that Conditions (3) and (7) do not contain dimensional constants.

BIBLIOGRAPHY

1. Leibenzon, L.S., Dvizhenie prirodnykh zhidkostei i gazov v poristoi srede (The motion of natural fluids and gases in a porous medium). GITTL, Moscow-Leningrad, 1947.

2. Barenblatt, G.I., O predel'nykh avtomodel'nykh dvizheniiakh v teorii

nestatsionarnoi fil'tratsii gaza v poristoi srede i teorii pogranichnogo sloia (On limiting self-similar motions in the theory of non-stationary filtration of a gas in a porous medium and in boundary layer theory). *PMM* Vol. 18, No. 4, 1954.

3. Sedov, L.I., Metody podobiia i razmernosti v mekhanike (Similarity methods and dimensionality in mechanics). GITTL, 1954.

Translated by R.D.C.